

## DOCUMENT RESUME

ED 463 140

SE 065 528

AUTHOR Picard, Gilles; Pineau, Kathleen  
TITLE Help in Using Symbolic, Hand-Held Calculators in Teaching Engineering Mathematics.  
PUB DATE 2001-06-00  
NOTE 8p.; Paper presented at the Annual Applications of Computer Algebra Conference (Albuquerque, NM, May 31-June 3, 2001).  
AVAILABLE FROM For full text:  
[http://www.seg.etsmtl.ca/gilles/conf/final\\_albuq.pdf](http://www.seg.etsmtl.ca/gilles/conf/final_albuq.pdf). For full text:  
[http://math.unm.edu/ACA/2001/Proceedings/Education/PicardPineau\\_paper.pdf](http://math.unm.edu/ACA/2001/Proceedings/Education/PicardPineau_paper.pdf).  
PUB TYPE Reports - Descriptive (141) -- Speeches/Meeting Papers (150)  
EDRS PRICE MF01/PC01 Plus Postage.  
DESCRIPTORS \*Engineering Education; \*Graphing Calculators; Higher Education; Mathematics Education; Teaching Methods

## ABSTRACT

Since the 1999 fall semester, the TI-92 Plus or the TI-89 has been a compulsory purchase for new students entering one engineering school in Montreal, Canada (almost 1000 enrolments a year). The purpose of this paper is to share impressions of a campus-wide experience and to communicate solutions adopted to help students and faculty members in the use of these calculators. (MM)

## Help in using Symbolic, Hand-held Calculators in Teaching Engineering Mathematics

Gilles Picard, Kathleen Pineau  
École de technologie supérieure  
Montréal, Canada

[gpicard@seg.etsmtl.ca](mailto:gpicard@seg.etsmtl.ca)

[kpineau@seg.etsmtl.ca](mailto:kpineau@seg.etsmtl.ca)

---

*Since the 1999 fall semester, the TI-92 Plus or the TI-89 has been a compulsory purchase for new students entering our engineering school (almost 1000 enrolments a year). The purpose of this paper is to share our impressions of this campus-wide experience and to communicate the solutions that we considered and adopted to help students and faculty members in the use of these calculators.*

---

### Introduction

To give you a perspective of our working environment, we teach undergraduate level courses at the École de technologie supérieure (ETS) in Montréal. It is a French technical engineering school where undergraduate student enrolment stands at approximately 2500.

Since the 1999 fall semester, the TI-92 Plus or the TI-89 has been compulsory for all new full-time students entering our engineering school. Both models offer the same symbolic computation capabilities; they differ mainly by their physical shape and size. Students learn to use their calculators during their first mathematics course, calculus.

The symbolic and algebraic computation capabilities of these machines have naturally brought us to question and change the *what* and the *how* we teach mathematics. Students now have a tool that can do in a few seconds what would take much longer when done manually. For example, there is quite a bit of calculation involved in the decomposition of this rational function into partial fractions.

$$\frac{t^3 - t + 1}{(t+5)(t+2)^2(t^2+4)} = \frac{-119}{261} + \frac{61}{144} + \frac{-5}{24} + \frac{15}{464} \frac{t-13}{t^2+4}$$

The concept of decomposition is simple enough, it is the amount of algebraic manipulations required that is time-consuming. We consider that once students have understood a concept, and that the subject can benefit from it, long and tedious manual calculations are better left to the calculator. This gives us extra time for the deeper exploration and understanding of the concept. Incidentally, new topics come up naturally when using these calculators. For example, see screen 1, when students are asked to input the function  $e^t + e^{-t}$  into the calculator<sup>1</sup> they look to us for an explanation.

---

<sup>1</sup> Our students do not usually know about hyperbolic functions beforehand.

July 9, 2001

PERMISSION TO REPRODUCE AND  
DISSEMINATE THIS MATERIAL HAS  
BEEN GRANTED BY

*G. Picard*

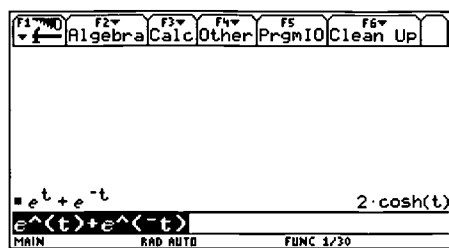
TO THE EDUCATIONAL RESOURCES  
INFORMATION CENTER (ERIC)

U.S. DEPARTMENT OF EDUCATION  
Office of Educational Research and Improvement  
EDUCATIONAL RESOURCES INFORMATION  
CENTER (ERIC)

This document has been reproduced as  
received from the person or organization  
originating it.

Minor changes have been made to  
improve reproduction quality.

• Points of view or opinions stated in this  
document do not necessarily represent  
official OERI position or policy.

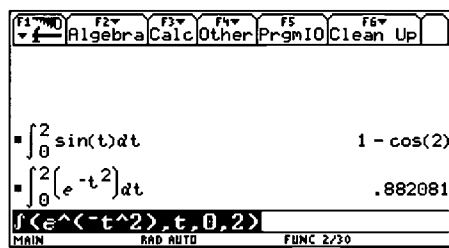


screen 1

### An in-class example

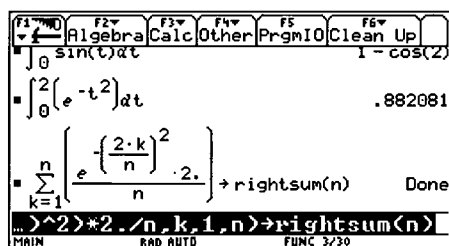
To illustrate how our lectures have changed with the introduction of these calculators, here is an example of one of our presentations. It is part of a 90-minute lecture.

When presenting the second fundamental theorem of calculus, we start by reviewing the first fundamental theorem of calculus, using the TI's computation capabilities. We know that when an anti-derivative of the function we wish to integrate is found, we can evaluate it at the upper and lower limits to get an exact answer. This is also what the calculator does; see the first line of screen 2. However, if no anti-derivative in terms of elementary functions exists or can be found, we must estimate the definite integral's value by approximation. This is also what the calculator does; see the second line of screen 2.



screen 2

We use this opportunity to review Riemann right-hand and/or left-hand sums by having the students program these sums on their machines; see screen 3.



screen 3

Now that all our students have their own TI, the computation of Riemann sums that was once long and tedious, is now quick and easy. Students cannot hide from the mathematical notation when using these calculators. They develop a better understanding of the mathematical notation involved because they have to use it! Also, when asked to evaluate such sums for different values of  $n$ , with  $n$  being the number of subintervals, they are always happy to shout out their answers. We can take down their answers to build a table of values for the different values of  $n$ . It is easy to see that the constructed sequence converges to the value of the definite integral we are looking for; see screen 4.

F1	F2	F3	F4	F5	F6
Algebra	Calc	Other	PrgmIO	Clean Up	
$\int_0^2 (e^{-t^2}) dt$					.882081
rightsum(25)					.842775
rightsum(50)					.862438
rightsum(100)					.872262
rightsum(250)					.878154
rightsum(500)					.880118
<b>rightsum(500)</b>					
MAIN			RAD AUTO		FUNC 7/30

screen 4

After reviewing the 1<sup>st</sup> fundamental theorem of calculus we introduce the 2<sup>nd</sup>, which states that the derivative of a definite integral with respect to its upper limit is the integrand at the upper limit. We start by performing the necessary calculations on the blackboard so students can understand the algebraic meaning of the 2<sup>nd</sup> fundamental theorem of calculus. Once this has been done, we can use the TI's symbolic computation capabilities to further develop their understanding of the theorem.

When there exists an anti-derivative in terms of elementary functions, such as for  $\sin(t)$ , the TI performs the same type of calculations that we do manually; see screen 5.

F1	F2	F3	F4	F5	F6
Algebra	Calc	Other	PrgmIO	Clean Up	
$\int_{\text{constant}}^x \sin(t) dt$					
					$\cos(\text{constant}) - \cos(x)$
$\frac{d}{dx}(\cos(\text{constant}) - \cos(x))$					$\sin(x)$
<b>d(ans(1),x)</b>					
MAIN			RAD AUTO		FUNC 2/30

screen 5

F1	F2	F3	F4	F5	F6
Algebra	Calc	Other	PrgmIO	Clean Up	
					$\cos(\text{constant}) - \cos(x)$
$\frac{d}{dx}(\cos(\text{constant}) - \cos(x))$					$\sin(x)$
$\int_0^x (e^{-t^2}) dt$					$\int_0^x (e^{-t^2}) dt$
$\frac{d}{dx} \left( \int_0^x (e^{-t^2}) dt \right)$					$e^{-x^2}$
<b>d(ans(1),x)</b>					
MAIN			RAD AUTO		FUNC 4/30

screen 6

On the other hand, when no anti-derivative in terms of elementary functions exists, such as for the Gaussian curve, the TI simply rewrites the integral; see screen 6. Since the definite integral

$$\int_0^x e^{-t^2} dt$$

is a function of  $x$ , the upper limit, we can expect it to have a derivative.

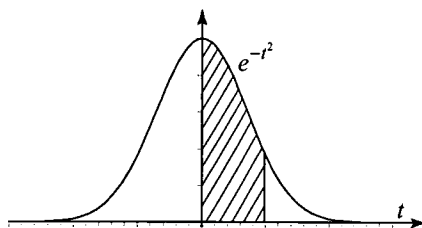


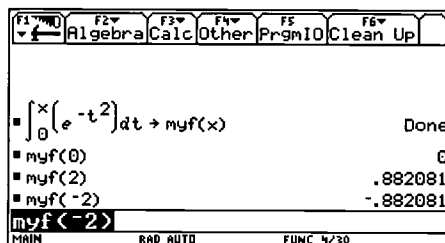
figure 1

By using the TI to compare what happens to the two different types of functions, one that has a simple anti-derivative and one that does not, we can easily put the emphasis on the fact that we don't need to find the anti-derivative to use the theorem, as illustrated on screen 6.

Many students have difficulties with functions and functional notation. With symbolic computation capabilities at hand, we can easily explore with our students, and in our classroom, the properties of the function defined as the following definite integral:

$$myf(x) = \int_0^x e^{-t^2} dt.$$

We start by defining the integral as  $myf(x)$  on the calculator, see screen 7, and ask students to use figure 1 to find  $myf(0)$  and only then, compare their answer with the TI's. We also have them confirm the earlier limit of Riemann sums by evaluating  $myf(2)$  and have them notice the busy signal that comes up on the calculator. The fact that it takes the machine a few seconds to evaluate  $myf(2)$  gives us the opportunity to remind students that it is proceeding by numerical approximation. Students get a better understanding of what is going on because they went through the approximation process beforehand. Finally, we also have students evaluate  $myf(-2)$  directly on the TI, expecting the same kind of answer as  $myf(2)$ , and ask them to explain why the answer is negative.



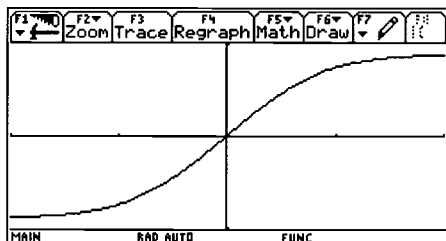
screen 7

The next natural step is to graph  $myf(x)$ . This can take some time on the TI. While the machine is evaluating and connecting the dots, we turn to the blackboard and figure out what the graph should look like. We know, by the 2<sup>nd</sup> fundamental theorem of calculus, that  $myf'(x) = e^{-x^2}$ . We can easily show

that  $myf'(x) > 0, \forall x \in \mathbb{R}$ , and that  $\lim_{x \rightarrow \pm\infty} myf'(x) = 0$ . Therefore  $myf(x)$  is increasing and, as  $x$  moves away from the origin, the curve flattens out. Also, since

$$myf''(x) = -2xe^{-x^2},$$

$myf(x)$  will be concave up when  $x < 0$ , concave down when  $x > 0$ , and there will be a point of inflexion at  $x=0$ . We take advantage of the TI's busy signals to answer student questions.



screen 8

The graphing process would be quicker on a computer. We used to use Maple or Derive, but students were passive in front of the display. We made the symbolic calculator compulsory because we want our students to use it in the classroom. We want all our students to actively participate in this process on their own calculator. As they try things, they tend to ask more questions and are generally more interested in what is happening. They also seem to have a better overall understanding of the material. The price we/they pay is that students develop less algebraic virtuosity. It is a price we are willing to pay to be able to tackle more challenging problems. Finally, it must be noted that we still have students use CAS (Computer Algebra Systems) on computers when printouts are needed or when the problems are better explored with more powerful computing tools.

### Mandatory calculators

Implementing the mandatory use of the TI-89/92 Plus calculators in the fall of 1999 wasn't an easy task. We had to make sure that the appropriate equipment was available and that all instructors were capable of using this equipment.

We realized that there were few resources available in French, on the web, to support the use of these calculators. Following our experience, in the summer 2000, we collected questions that students or instructors often asked us about the use of these calculators. We then designed (with the assistance of two colleagues, Michel Beaudin and Chantal Trottier) the web site <http://www.seg.etsmtl.ca/ti/>, which provides quick and easy access to information on the use of these calculators. Its guided tour, FAQ, and

different documents were designed to meet the specific needs of our students and faculty members. We have translated part of our web site into English, see <http://www.seg.etsmtl.ca/ti/English/>, to give you an idea of the work that was accomplished; figure 2 is a screen shot of the welcome page.

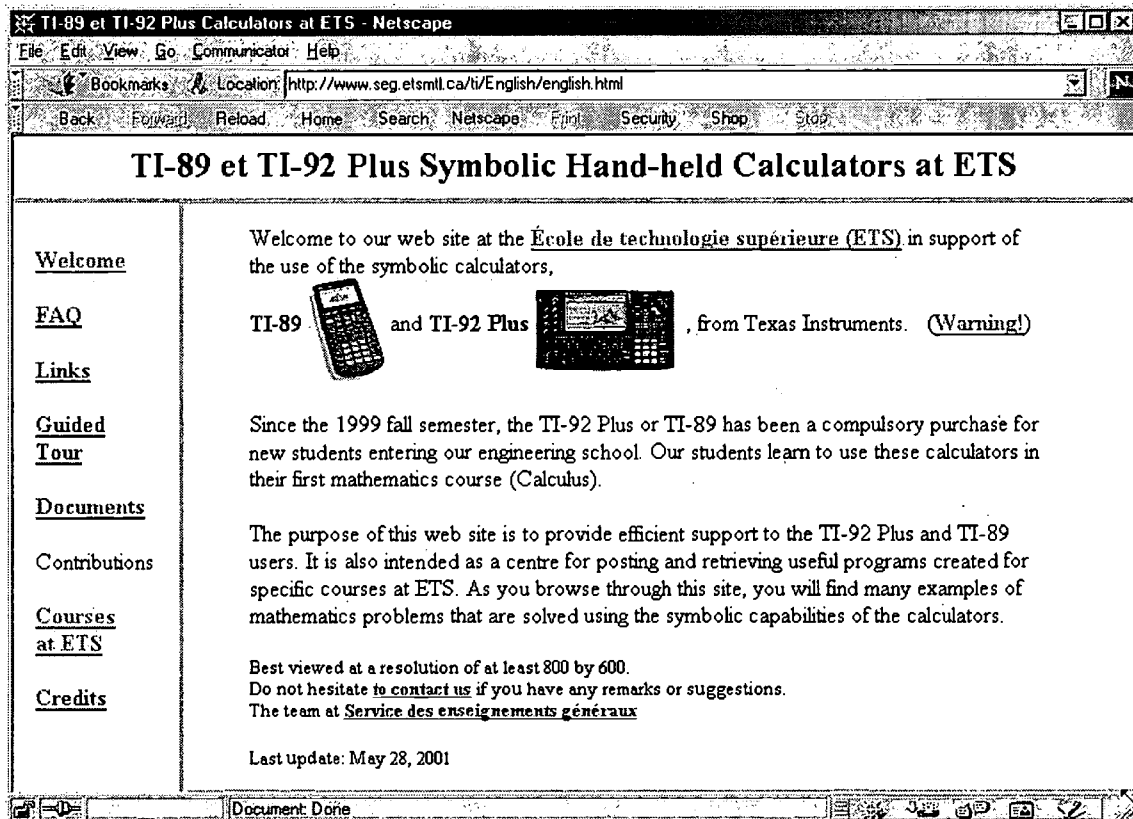


figure 2

The questions that we collected were classified into two groups. If a short answer was all that was necessary, it was put in the **FAQ** section. For instance, *How do I stop a calculation?* is listed in the **FAQ**. If the question demanded a more detailed explanation, such as *How do I solve a system of equations?* or *How do I enter and use matrices?* it was put in the **Documents** section of our site.

Another aspect of our experience was extending the use of these calculators to more courses at our university. We met with interested professors from the four engineering departments to present the calculator's potential, gave workshops, and informed them that their students had this technology at hand. To help stimulate discussion about this technology within the departments, and the changes it would inevitably bring to teaching, we hired four students, one per department, to work on small TI application modules. Each of these modules was designed to meet the needs of one or two courses in each of the four

departments. For example, under the supervision of two professors from the Civil Engineering department, a student produced the links associated to **CTN-208** and **CTN-408** (courses on Analysis of Structures and Mechanics of Materials) in the **Courses at ETS** page of our site. His job was to write a set of TI programs and/or commands that can be downloaded from our site onto the TI-89/92 Plus. He also had to produce the corresponding user's guide. Now, CTN-208 and CTN-408 students download these files to their TIs and use them in these courses.

### **Conclusion**

As the introduction of this hand-held technology forced us to re-assess our goals in teaching mathematics, it also stimulated many discussions among faculty members. What do we want our students to learn? What is important? In particular, what should students still be able to do manually?

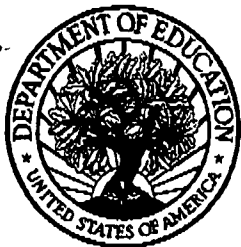
The hands-on approach made available by this technology makes for a more dynamic classroom as the students participate more actively in the exploration of mathematical ideas. We see using the available technology as one way to help students learn.

So far the results have been positive, students and most of our mathematics instructors are adapting to this new technology. There's still resistance from some faculty members and we must continue to put forward the benefits of integrating computer algebra in our teaching.

**Acknowledgments** We wish to thank Claude Pineau for proofreading this paper. Thanks are also due to Michel Beaudin and Chantal Trottier for their assistance in designing the web site [www.seg.etsmtl.ca/ti/](http://www.seg.etsmtl.ca/ti/).



05528



**U.S. Department of Education**  
 Office of Educational Research and Improvement (OERI)  
 National Library of Education (NLE)  
 Educational Resources Information Center (ERIC)



**REPRODUCTION RELEASE**  
 (Specific Document)

**I. DOCUMENT IDENTIFICATION:**

Title: <u>HELP IN USING SYMBOLIC, HAND-HELD CALCULATORS IN TEACHING ENGINEERING MATHEMATICS</u>	
Author(s): <u>PICARD, G PINEAU, K.</u>	
Corporate Source:	Publication Date: <u>AUGUST 2001</u>

**II. REPRODUCTION RELEASE:**

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education* (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic media, and sold through the ERIC Document Reproduction Service (EDRS). Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following three options and sign at the bottom of the page.

The sample sticker shown below will be affixed to all Level 1 documents

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

GILLES PICARD

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

1

Level 1

X

Check here for Level 1 release, permitting reproduction and dissemination in microfiche or other ERIC archival media (e.g., electronic) and paper copy.

The sample sticker shown below will be affixed to all Level 2A documents

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE, AND IN ELECTRONIC MEDIA FOR ERIC COLLECTION SUBSCRIBERS ONLY, HAS BEEN GRANTED BY

\_\_\_\_\_

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

2A

Level 2A

X

Check here for Level 2A release, permitting reproduction and dissemination in microfiche and in electronic media for ERIC archival collection subscribers only

The sample sticker shown below will be affixed to all Level 2B documents

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE ONLY HAS BEEN GRANTED BY

\_\_\_\_\_

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

2B

Level 2B

X

Check here for Level 2B release, permitting reproduction and dissemination in microfiche only

Documents will be processed as indicated provided reproduction quality permits. If permission to reproduce is granted, but no box is checked, documents will be processed at Level 1.

I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries.

Sign here, please

Signature: <u>[Signature]</u>	Printed Name/Position/Title: <u>GILLES PICARD</u>	
Organization/Address: <u>ECOLE DE TECHNOLOGIE SUPERIEURE 1100 rue NOTRE-DAME OUEST MONTREAL CANADA H3C 1K3</u>	Telephone: <u>(514) 396-8500</u>	FAX: <u>(514) 396-8500</u>
	E-Mail Address: <u>GILLES.PICARD@ETSMTL.CA</u>	Date: <u>2008/04/08</u>